Management Of The Recolonisation Of Rhinos

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Abstract

The purpose of this investigation is to develop a suitable model to describe harvesting of black rhinos in South Africa. This model aims to assist in managerial decisions in order to help increase the population of black rhinos in South Africa. In particular, harvesting of black rhinos between the Kruger National Park and HIB park will be studied.

Introduction

With both black and white rhino populations presently at such low levels, conservation now more than ever, takes a prominent role. With a current population of black rhinos at approximately 2000, the monitoring and relocation of these rhinos is paramount. The present day rhino faces an array of challenges, in particular, the South African rhino is under increasing threat from illegal killings(poaching) due to their horns being in such high demand.

Although the black rhino is not as sort after as the white rhino by poachers, its low numbers force us to pay particular attention to its conservation. This paper aims to investigate how best one should conduct rhino removals with regards to space limitation as well as gender in an attempt to maximise the reproduction potential of the total population.

Aims

In order to maximise this potential, many factors and parameters need to be considered. The primary objective of this problem is to study the frequency and location of rhinos to be moved. Influencing factors include the age, sex and size of the rhinos. Furthermore, the density of the population is of critical importance. It has been found that the relocation of rhinos from high density to low density populations increases the overall population growth rate. This Study aims to optimise this process. Harvesting, the removal and relocation of rhinos, should be conducted at a rate which ensures that what is left in one park is sufficient to increase its reproduction rate and thus its overall population.

Since rhinos are territorial animals, relocated rhinos may behave in unpredictable ways. For example, there is no guarantee that relocated rhinos will claim new territories as their own and this may hinder overall population growth.

Although gender is an influencing factor, inclusion of it into the model developed was beyond the scope of this workshop.

Single Population Model

We begin with a single population model in order to understand the dynamics involved. One of the most commonly used models is the logistic equation,

$$\frac{dP}{dt} = R(t)P(t)\left(1 - \frac{P(t)}{K(t)}\right), \qquad P(0) = P_0 \quad (1)$$

where $K(t)$ is the carrying capacity[1], $R(t)$ is the growth rate, with $P(t)$ is the population at time t and P_0 is the initial population at time $t = 0$.

We non dimensionalise the equation by introducing the variables,

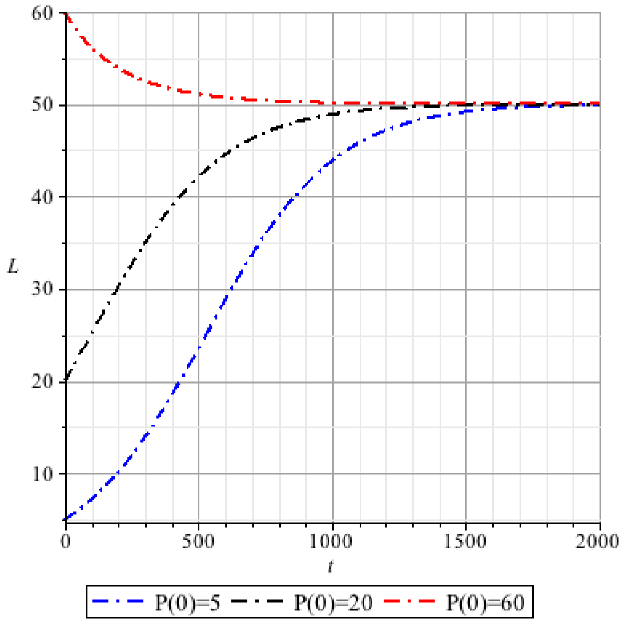
$$\bar{P} = \frac{P}{K_0}, \quad \bar{R} = \frac{R}{R_0}, \quad \bar{K} = \frac{K}{K_0}, \quad \bar{T} = TR_0, \quad (2)$$

where K_0, R_0 are representative values for the functions K and R. When we non dimensionlise we suppress the bar for simplicity in the resultant equations.

Using equation (2) and supposing that the carrying capacity and growth rate is constant, we obtain,

$$\frac{dP}{dt} = P(1-P), \qquad p(0) = \frac{P_0}{K_0}. \qquad (3)$$

Figure 1: *Logistic Model*



We can see in Figure 1 that the initial populations below and above the carrying capacity tend asymptomatically towards it.

Allee Effect

The Allee effect is a phenomenon found in biology. In this case, the Allee effect represents a lower bound for our initial population of rhinos. Any initial population found below this value will tend to extinction and have no possibility towards positive growth.

Considering the Allee effect in non dimensionalised form, the logistic equation becomes,

$$\frac{dP}{dt} = P(1-P)(\alpha P - 1), \qquad P(0) = P_0, \qquad (4)$$

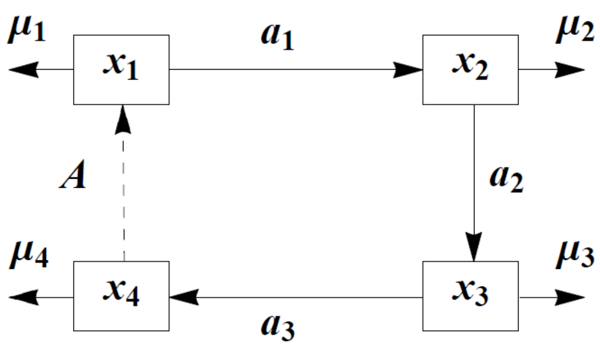
where α denotes the Allee parameter.

We are not sure whether the Allee effect has an effect on the rhino problem but in general it does have an effect on populations.

Age Structure

In Figure 2, we denote x_1 as the population of newborn rhinos, x_2 as infant rhinos, x_3 sub-adult rhinos and x_4 the adult population of rhinos. Further, a_1, a_2, a_3 denotes the survival rate between sub-populations and $\mu_1, \mu_2, \mu_3, \mu_4$ the death rate respectively. A represents the birth rate. Each

Figure 2: *Single Population Model*



compartment relates to various age categories: newborns or unweaned rhinos found in the x_1 group range from 0 to 1 years. Infant or juveniles found in x_2 cover years 1 through 2, while the sub adults found in x_3 pertain to rhinos ages 2 to 3. Rhinos aged more than 3 will be situated in the x_4 group.

The infant age group x_1 represents rhinos that are still dependent on their mothers, x_2 represents rhino that are less dependent on their mothers but do not stray too far. The rhinos in x_3 are now independent but not yet able to reproduce and the reproductive age group is represented by x_4.

Harvesting of female rhinos takes place in the

[1] The carrying capacity is the maximum number of animals an environment can sustain.

sub adult range to ensure that the possibility of pregnancy is low, and male rhinos are harvested from x_4 as they have attained their maximum weight and thus able to defend themselves against other male rhinos.

This age structure can be modelled using the following ODEs,

$$\frac{dx_1}{dt} = Ax_4 - x_1(a_1 + \mu_1), \tag{5}$$

$$\frac{dx_2}{dt} = a_1x_1 - x_2(a_2 + \mu_2), \tag{6}$$

$$\frac{dx_3}{dt} = a_2x_2 - x_3(a_3 + \mu_3 + h_3), \tag{7}$$

$$\frac{dx_4}{dt} = a_3x_3 + \epsilon x_4, \tag{8}$$

where $\epsilon \ll 1$ is our stability parameter. Due to time constraints, we do not solve this system as a result of muti-scaling requirement. We do however, perform a stability analysis on the system. Let,

$$\begin{aligned} a_1 + \mu_1 &= b_1, \\ a_2 + \mu_2 &= b_2, \\ a_2 + \mu_2 + h_3 &= b_3, \\ a_3 &= 0. \end{aligned} \tag{9}$$

written the above in matrix form we have the following,

$$\begin{pmatrix} -b_1 & 0 & 0 & A \\ a_1 & -b_2 & 0 & 0 \\ 0 & a_2 & -b_3 & 0 \\ 0 & 0 & 0 & \epsilon \end{pmatrix}$$

.

When ϵ is positive we get eigenvalues with a positive real entry which implies that the system is unstable.

When ϵ is negative, all eigenvalues were negative entries which implies that the system is stable and when $\epsilon = 0$ we have one eigenvalue with a zero entry which implies that the system behaves as an undamped oscillator.

Two Population Model

In this model, we neglect the problematic fact of age structure in a population. We consider two populations, whose rate of change in animal numbers are both described by the logistic equation. Define P_1 and P_2 to be populations 1 and 2 respectively. We firstly assume that the initial population $P_1(0)$ is at least twice as large as $P_2(0)$. The governing equations for P_1 and P_2 are,

$$\frac{dP_1}{dt} = \mu_1 P_1(1 - \frac{P_1}{K_1}) - hP_1, \tag{10}$$

$$\frac{dP_2}{dt} = \mu_2 P_2(1 - \frac{P_2}{K_2}) + \alpha h P_1, \tag{11}$$

where μ_1, μ_2 are growth rates of P_1 and P_2 respectively. K_1, K_2 are the carrying capacities while h represents the harvesting rate and α the number of successfully harvested rhinos.

The non-dimensionalised governing equations and the initial conditions for P_1 and P_2 are given by,

$$\frac{dP_1}{dt} = P_1(1 - P_1) - \sigma P_1, \qquad P_1(0) = P_{01}, \tag{12}$$

$$\frac{dP_2}{dt} = rP_2(1 - \frac{P_2}{\beta}) + \sigma n P_1, \quad P_2(0) = P_{02}, \tag{13}$$

where r is the ratio of the growth rate in population P_1 to the growth rate in population P_2. σ is the fraction of animals harvested from P_1, n the fraction of rhinos that are successfully integrated into P_2 and β is the ratio of the carrying capacity in P_1 to that of P_2.

The following assumptions are made:

1. $0 < \sigma < \frac{1}{4}$
   We assume that rhinos are taken from population 1. We also assume that not more than a $\frac{1}{4}$ of the total population of rhinos in P_1 are harvested.

2. $\beta < 1$
   Here we assume that the region in which population P_1 is situated has a greater carrying capacity than that of the region where population 2 is situated.

3. $0 \leq$ n ≤ 1
   This means that the fraction of animals harvested from population 1 and are successfully integrated into population 2 lies between 0 and 1.

4. $0 \leq$ r ≤ 2
   The growth rate in P_1 is assumed to be at most twice the growth rate in population 2.

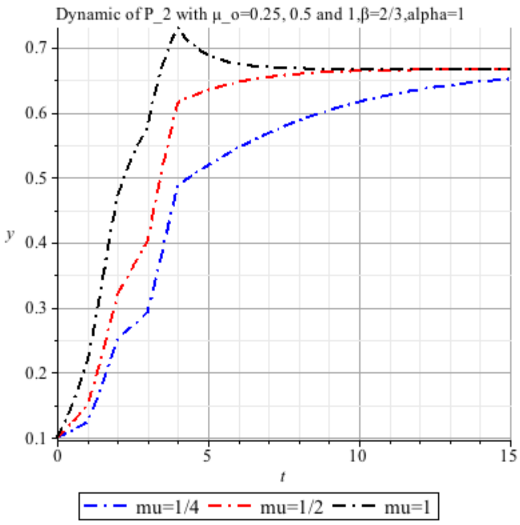
The following plots reveal the behaviour of these populations for various values of the parameters.

Discussion

The graphs of population P_2 in Figure 3 compare different values for the ratio of the growth rate in population 1 to the growth rate in population 2. An optimal rate exists, which will enable this population to reach the equilibrium point (while remaining below it) in the minimum amount of time. We denote σ as the harvesting rate over the growth rate of population 1. Consider the following values,

$$\begin{cases} \sigma = 0.2 & : \quad 1 < t < 2 \\ \sigma = 0.2 & : \quad 3 < t < 4 \\ \sigma = 0 & \text{for all other years} \end{cases}$$

Figure 3: *Population P_2*



The graphs of P_2 are shown in Figure 4. These graphs have different values of the parameter α and from this we can see that more human protection from harvested rhinos assists in increasing the population growth. However, in the long term, this parameter does not seem to be important.

Graphs of populations P_1 (x_1) and P_2 (x_2) are shown in Figure 5. These graphs look at a time range from 0 to 5 years, i.e the short term effect of harvesting. The decrease in the population growth of the harvested population is counteracted by a larger positive increase in the recipient population. Two different values for μ_1 were used namely, 1/2 and 1/3. In graphs 5 and 6, the following values were use for the other parameters, $\mu_2 = 1/4, K_1 = 60, K_2 = 50, \beta = 0.75$, while $h = \sigma$ where σ is defined above. The graphs in Figure 6 differ from those in Figure 5 in that a longer time

Figure 4: *Population P_2*

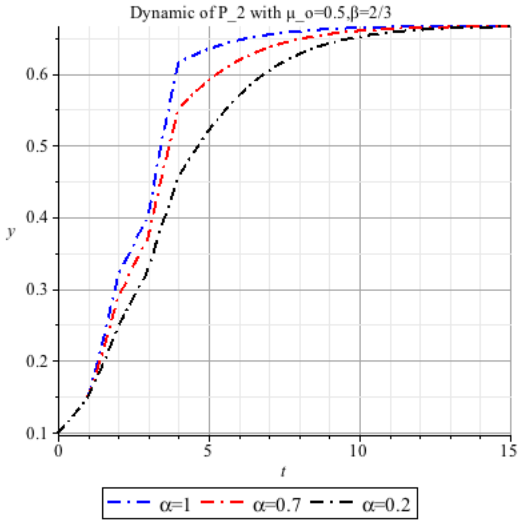
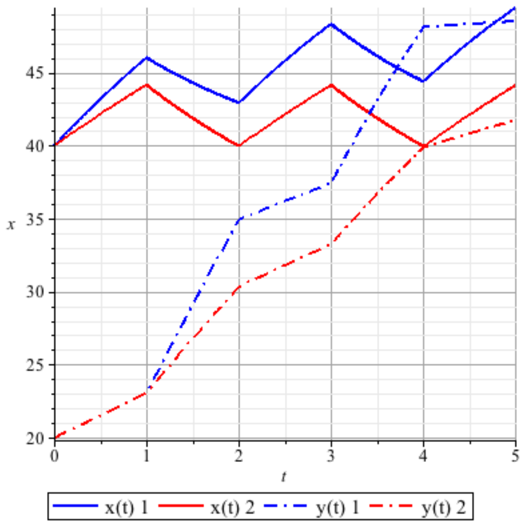


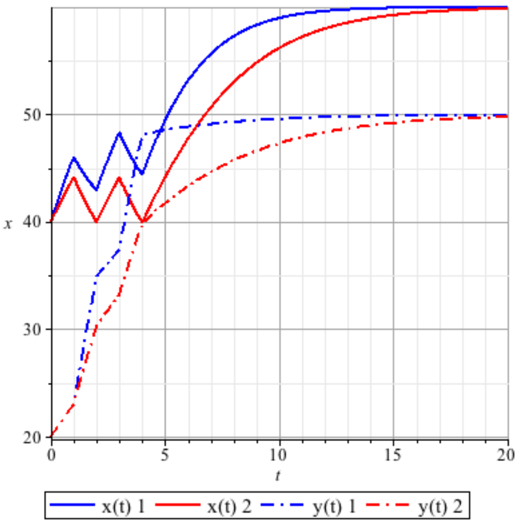
Figure 5: *Populations P_1 and P_2*



period is considered.

In both cases for the different values of μ_1,

Figure 6: *Populations P_1 and P_2*



harvesting for population 1 does not cause permanent damage to this population. A positive increase was seen in the curves of population 2.

Conclusion

Harvesting from the parent population must be carried out in a way that does not permanently affect the growth rate of this population. Harvesting must have a positive impact on the growth rate of the recipient population. In this report we compared the effect of changing different parameters on both of the populations. From our results we deduced that harvesting twice in consecutive 3 year time periods does not permanently affect the parent population and with time the population numbers will recover and tend to equilibrium. The recipient's population growth rate increased as a result of harvesting and also tended to a stable value. During harvesting, the parent population numbers decreased but this was counteracted by an even greater positive increase in the recipient population. Harvesting has a positive impact on both populations; however, it must not be carried too frequently as this would negatively affect the two populations.

References

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